GRAFIEK VAN EEN FUNCTIE

Asymptoten van functies $f(x)$

Verticale $x = a \quad \Leftrightarrow \lim\limits_{x \overset{>}{\to} a} f(x) = \infty \quad$ of $\quad \lim\limits_{x \overset{<}{\to} a} f(x) = \infty \quad$ ofwel beide

Horizontale $y = b \quad \Leftrightarrow \lim\limits_{x \to +\infty} f(x) = b \quad$ of $\quad \lim\limits_{x \to -\infty} f(x) = b \quad$ ofwel beide.

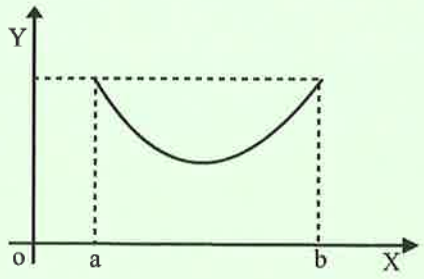
Schuine $y = ax + b \quad a = \lim\limits_{x \to \pm\infty} \dfrac{f(x)}{x} \qquad b = \lim\limits_{x \to \pm\infty} [f(x) - ax]$

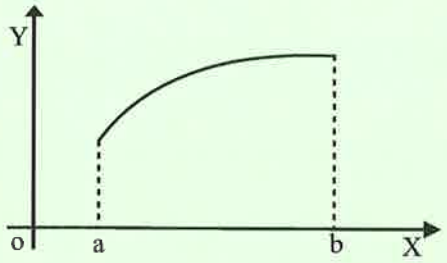
Stijgen en dalen

$\left.\begin{array}{l} f'(a) > 0 \\ f'(a) = +\infty \end{array}\right\} f(x)$ stijgend in $x = a$

$\left.\begin{array}{l} f'(a) < 0 \\ f'(a) = -\infty \end{array}\right\} f(x)$ dalend in $x = a$

Concaviteit, convexiteit en buigpunten van functies





$f(x)$ convex in $[a, b] \Leftrightarrow$
$f'(x)$ stijgend in $[a, b] \Leftrightarrow f''(x) > 0$ in $[a, b]$

$f(x)$ concaaf in $[a, b] \Leftrightarrow$
$f'(x)$ dalend in $[a, b] \Leftrightarrow f''(x) < 0$ in $[a, b]$

$f(x)$ buigpunt in $x = a \Leftrightarrow f''(x)$ verandert van teken in $x = a$

Uitgebreid extremumonderzoek van $f(x)$

$f'(a) = f''(a) = \cdots = f^{(n-1)}(a) = 0 \qquad f^{(n)}(a) \neq 0$
$\Rightarrow f$ extremum in $x = a \Leftrightarrow n$: even
$f^{(n)}(a) > 0$ Min $\qquad f^{(n)}(a) < 0$ Max

Uitgebreid buigpuntenonderzoek van $f(x)$

$f''(a) = f'''(a) = \cdots\cdots = f^{(n-1)}(a) = 0 \qquad f^{(n)}(a) \neq 0$
$\Rightarrow f(x)$ buigpunt in $x = a \Leftrightarrow n$: oneven